Properties of Some Common Geometric Shapes

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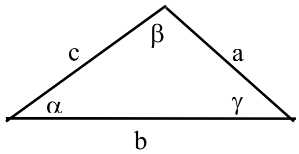
1. PLANE GEOMETRY

A collection of useful properties of triangles , quadrilaterals , polygons , and circles .

Triangles

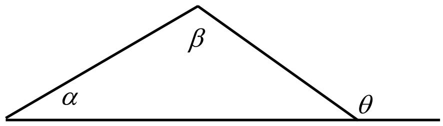
$$\alpha + \beta + \gamma = 180°$$

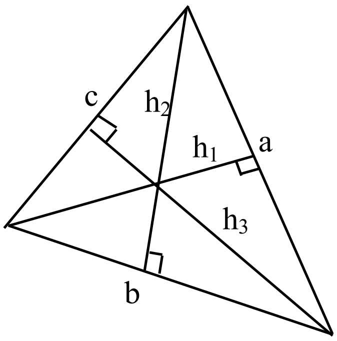
$$\alpha > \beta > \gamma \iff a > b > c$$

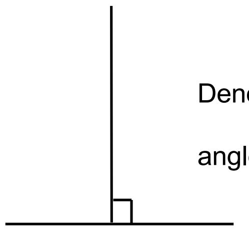


θ is called an exterior angle.
An exterior angle is equal to the sum of the two oposite interior angles

$$\theta = \alpha + \beta$$







Denotes an angle of 90°

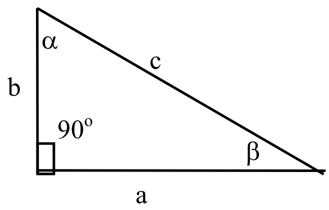
$$\text{Area} = \frac{1}{2}ah_1 = \frac{1}{2}bh_2 = \frac{1}{2}ch_3$$

Right-angled Triangles

If one of the angles is 90° the triangle is called a **right-angled triangle**

$a^2 + b^2 = c^2$ (Pythagoras Theorem)

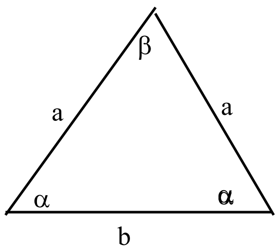
and Area = $\frac{1}{2}ab$.



Isosceles Triangles

If there are two angles equal it is an **isosceles triangle** and the sides opposite to the equal angles are also equal.

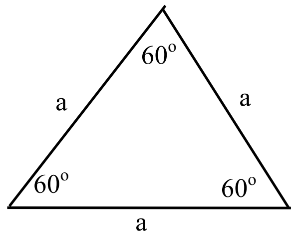
Area = $\frac{1}{4}b\sqrt{4a^2 - b}$



Equilateral Triangles

If $\alpha = \beta = \gamma = 60°$ the triangle is called an **equilateral triangle**, and all three sides are of equal length.

Area = $\dfrac{a^2\sqrt{3}}{4}$

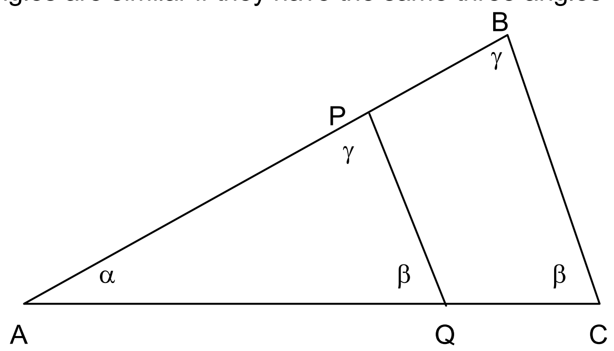


Congruent Triangles

Two triangles are congruent if they are identical apart from orientation

Similar Triangles

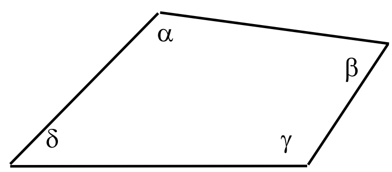
Two triangles are similar if they have the same three angles



The ratios of the corresponding sides are equal:- $\dfrac{BC}{PQ} = \dfrac{AB}{AP} = \dfrac{AC}{AQ}$

Quadrilaterals (Four sides)

$$\alpha + \beta + \gamma + \delta \quad = \quad 360°$$

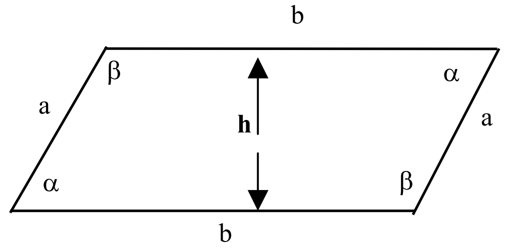


Parallelogram

A quadrilateral with opposite angles and opposite sides equal and $\alpha + \beta = 180°$ is called a **parallellogram.**

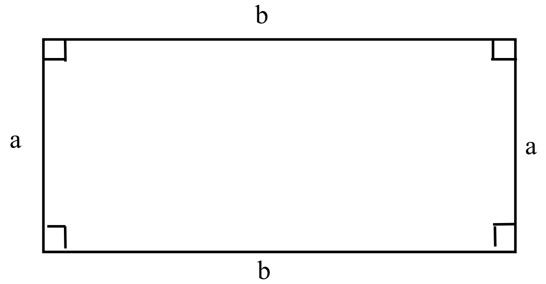
$$\alpha + \beta = 180°$$

Area **=** bh = absin α = absin β



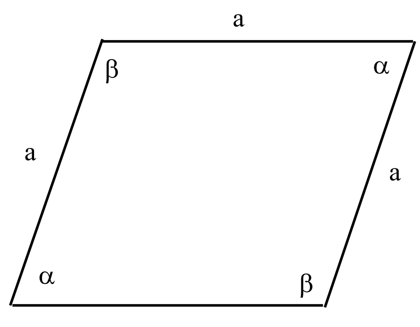
If $\alpha = \beta = 90°$ the figure is called a **rectangle**

Area = ab



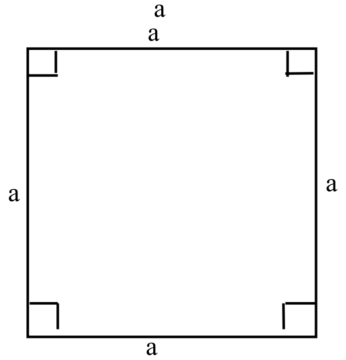
A parallelogram with all sides equal is called a **rhombus .**

Area = a^2sin α = a^2sin β



If $\alpha = \beta = 90°$ the figure is called a **square**.

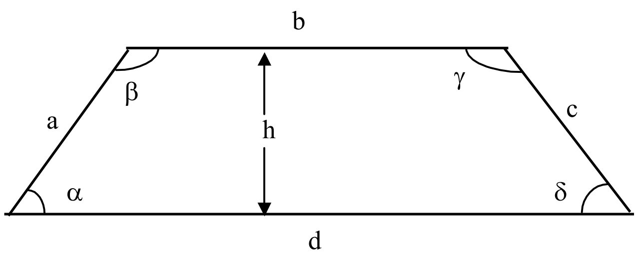
Area = a^2



Trapezium .

$\alpha + \beta = 180°$ and $\gamma + \delta = 180°$
Called a **trapezium** (or trapezoid)

Area = $\frac{1}{2}(b+d)h$



Polygons

Any plane closed figure having *n* straight sides (*n* corners) is called a **polygon**.

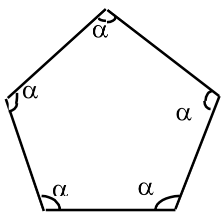
sum of angles = $(n-2)\cdot 180°$

If all the angles (and sides) are equal it is called a **regular polygon**

Angle = $\frac{n-2}{n}\cdot 180°$

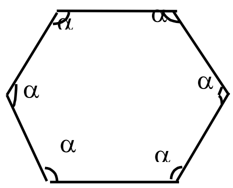
Examples of regular polygons

Pentagon



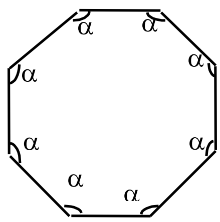
$\alpha = 108°$

Hexagon



$\alpha = 120°$

Octagon



$\alpha = 135°$

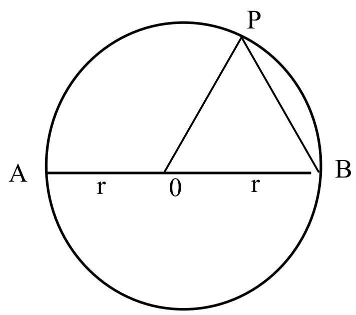
Circles

Circle with centre at 0.

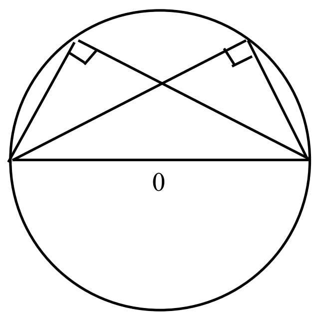
Radius $OP = r$
Diameter $AB = 2r = d$
Circumference $= 2\pi r = \pi d$

Area = $\pi r^2 = \frac{1}{4}\pi d^2$

Straight line PB is called a chord.

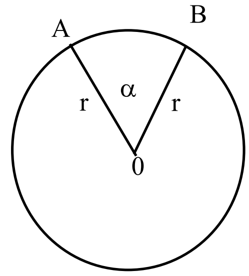


Any triangle inscribed in a semi-circle is a right angled triangle.



Circular Measure (Radians)

Arc length $AB = r\alpha$



Area of **sector** $OAB = \frac{1}{2}r^2\alpha$

where α is measured in radians.

1 radian $\equiv \frac{180}{\pi}$ degrees

1 degree $\equiv \frac{\pi}{180}$ radians.

Examples

$10^\circ \equiv \frac{10 \times \pi}{180} = 0.174533$ radians

$180^\circ \equiv \pi$ radians

$90^\circ \equiv \frac{\pi}{2}$ radians

$60^\circ \equiv \frac{\pi}{3}$ radians

$30^\circ \equiv \frac{\pi}{6}$ radians

$45^\circ \equiv \frac{\pi}{4}$ radians

1 radian $\equiv$ 57.2958 degrees.

2. SOLID GEOMETRY

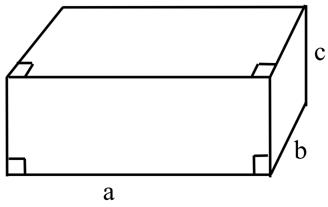
A collection of useful properties of parallelepipeds , pyramids , cylinders ,cones and spheres .

Rectangular parallelepiped

6 rectangular faces

Surface area = 2(*ab*+*ac*+*bc*)

Volume = *abc*

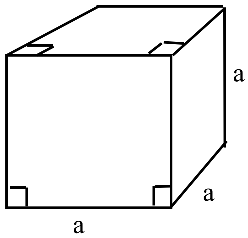


If all faces are square it is called a cube.

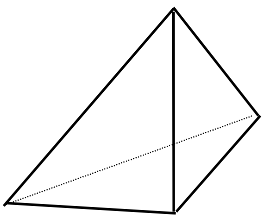
6 square faces

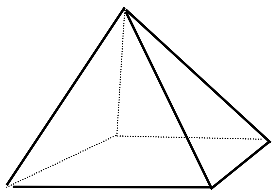
Surface area = $6a^2$

Volume = a^3



Pyramids





B = base area, h = height

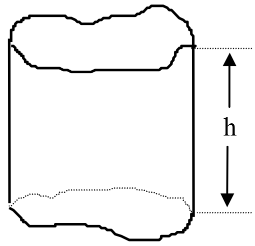
Triangular base

Volume = $\frac{1}{3}Bh$

Rectangular base

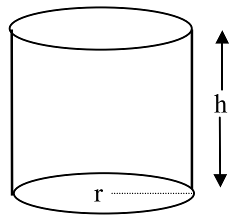
Volume = $\frac{1}{3}Bh$

Cylinders



General cylinder

Volume = Bh

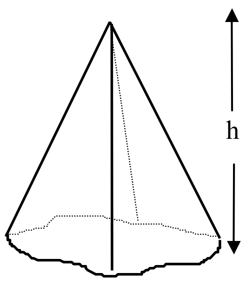


Right circular cylinder

Volume = $\pi r^2 h$

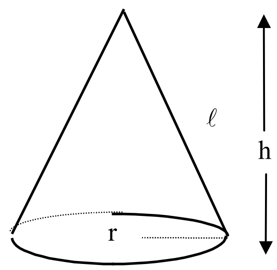
Curved surface area = $2\pi rh$

Cones



General cone

Volume = $\frac{1}{3}Bh$



Right circular cone

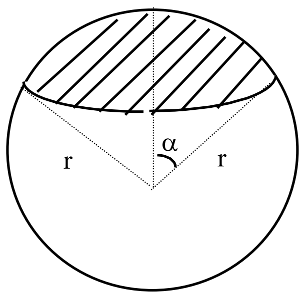
Volume = $\frac{1}{3}\pi r^2 h$

Curved surface area = $\pi r\ell$

Sphere

Surface area = $4\pi r^2$

Volume = $\frac{4}{3}\pi r^3$



Surface area of shaded cap = $4\pi r^2 \sin^2 \frac{\alpha}{2}$.

The quantity $4\pi \sin^2 \frac{\alpha}{2}$ is called the solid angle subtended by the cap at the centre of the sphere.